

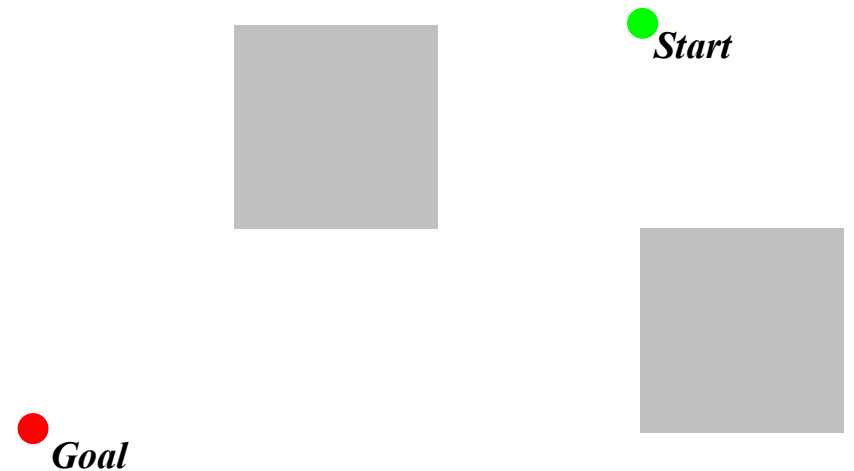
# Any-Angle Search

- Do not restrict paths to a grid or an a-priori given graph to find optimal paths in continuous environments

# Any-Angle Search



[from JPL]



- A. Nash and S. Koenig. Any-Angle Path Planning. Artificial Intelligence Magazine, 34(4), 85-107, 2013.

# A\* on Visibility Graphs

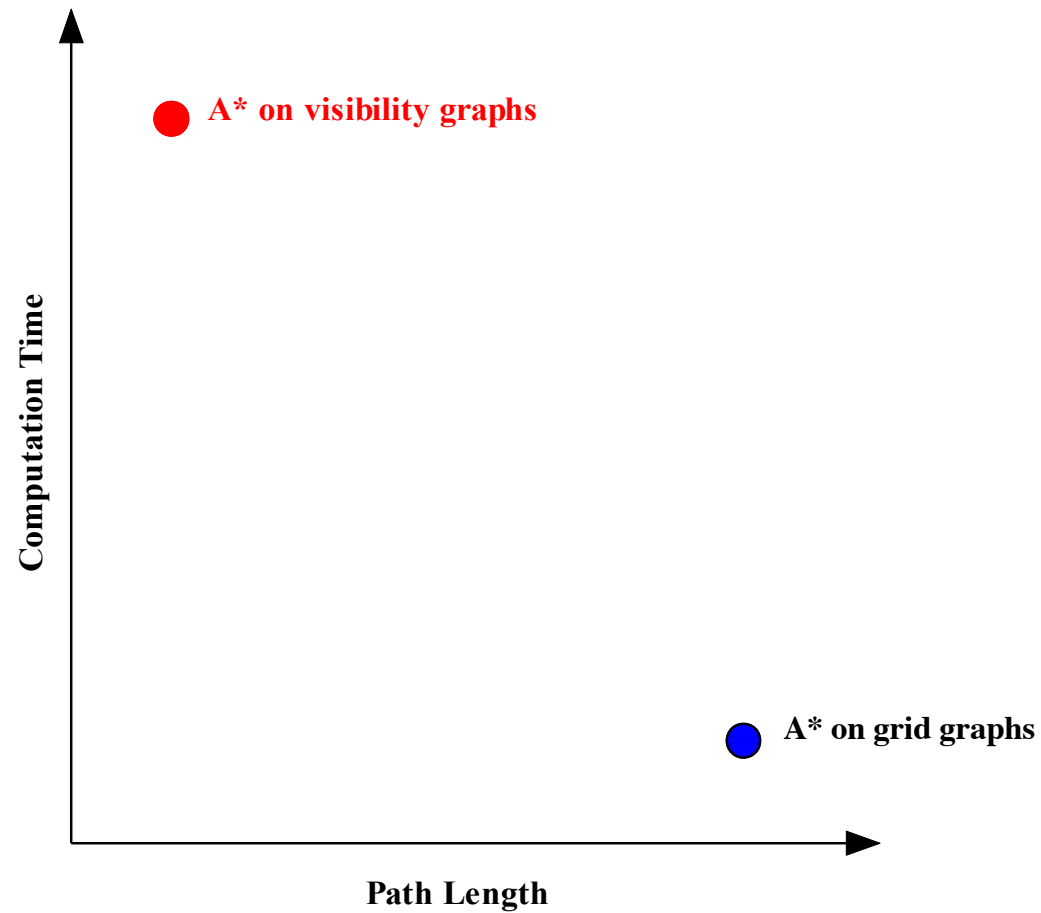
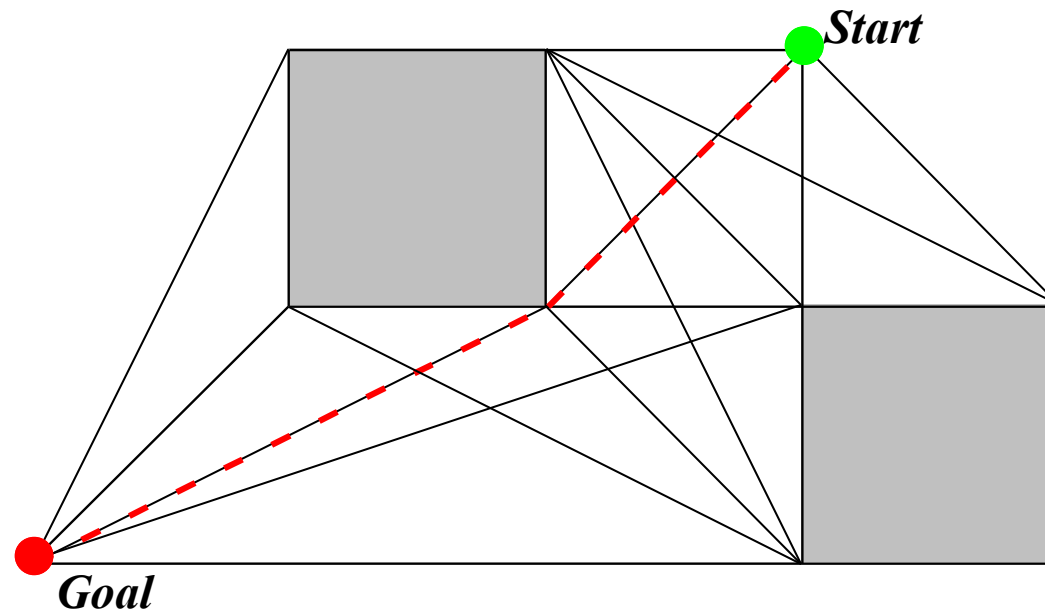


figure is notional

# A\* on Visibility Graphs

- A\* on Visibility Graphs [Lozano-Perez et al.]
- Note: Sophisticated versions exist, e.g. [Shah and Gupta]



- Shortest path in 2D terrain
- Slow due to many edges and line-of-sight checks

# A\* on Grid Graphs

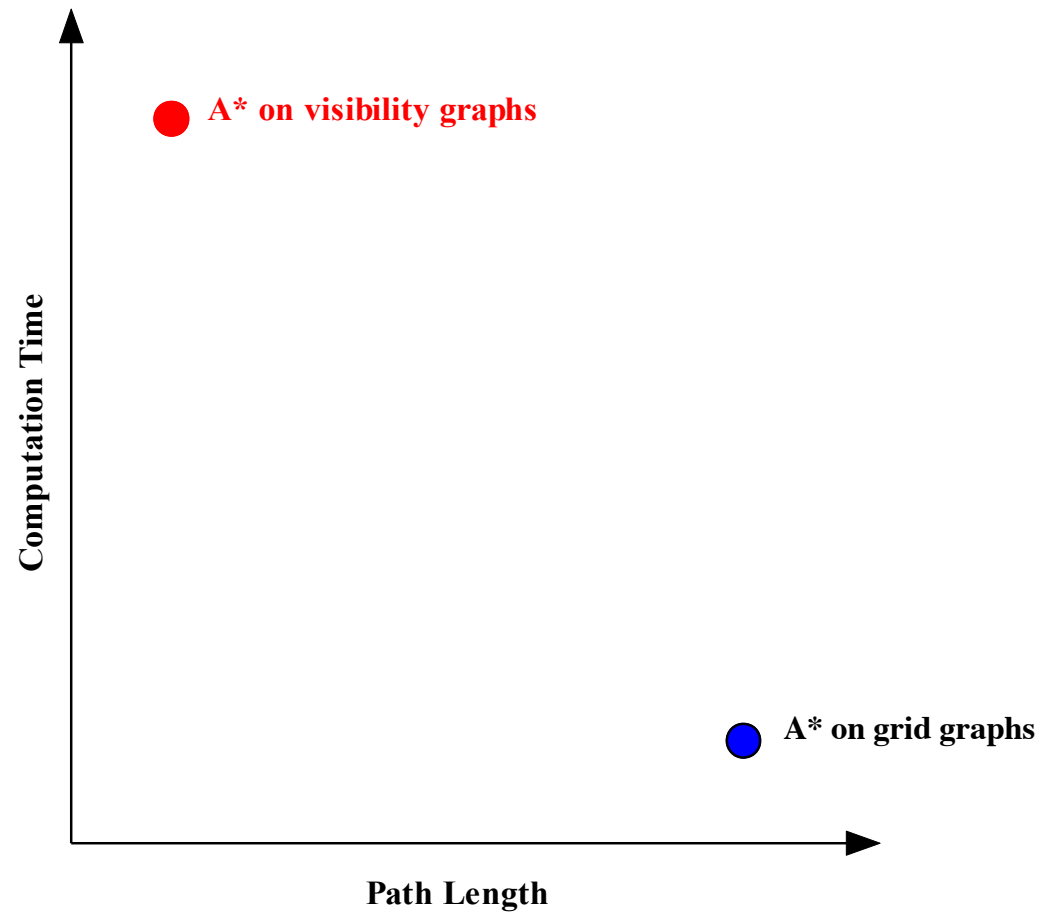
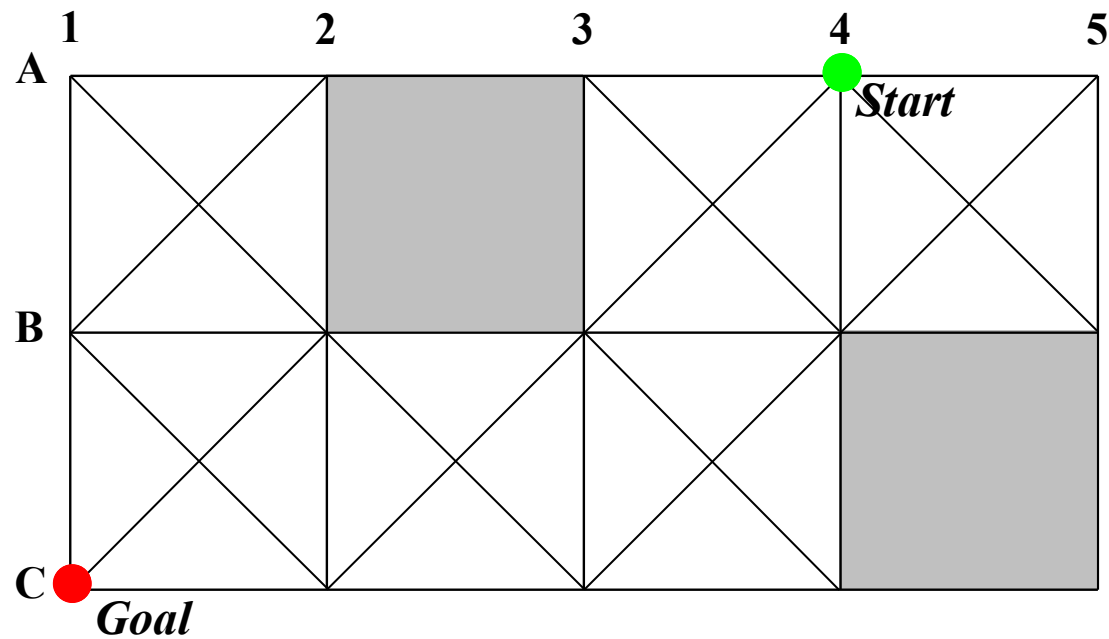


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# A\* on Grid Graphs

- A\* on grid graphs

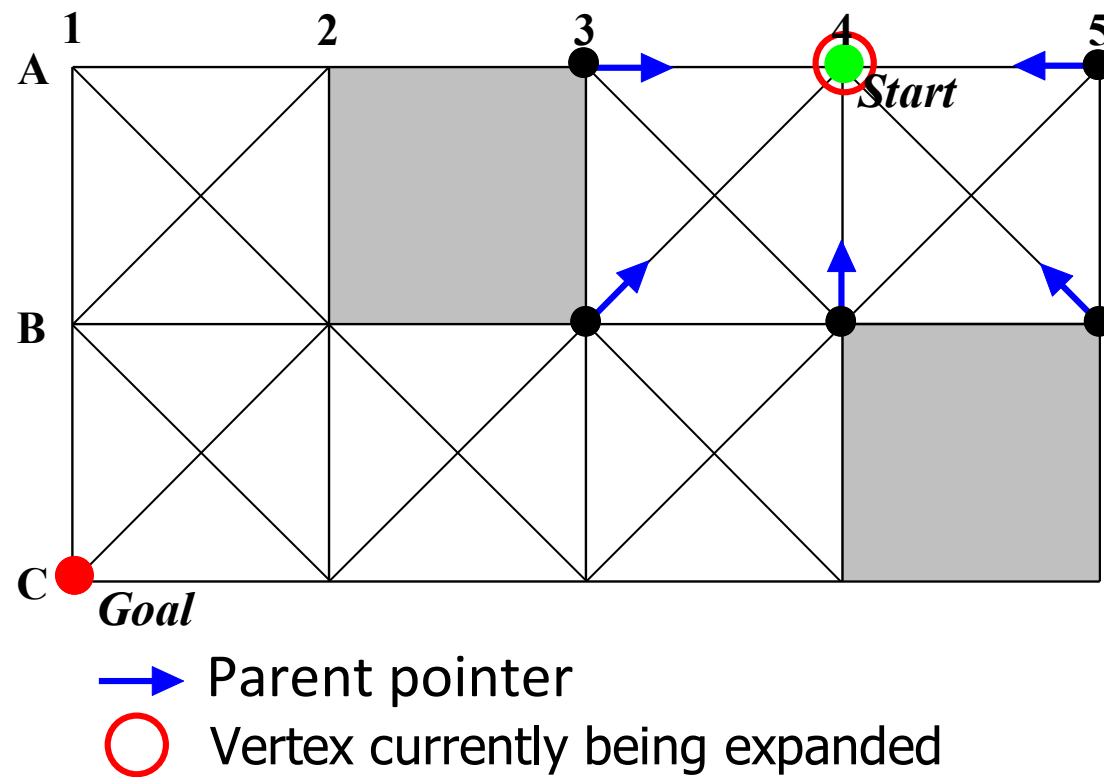


- A\* assigns two values to every vertex  $s$ 
  - $g(s)$ : the length of the shortest path from the start vertex to  $s$  found so far
  - $parent(s)$ : the parent pointer used to extract the path after termination
  - Following the parents from  $s$  to the start vertex results in a path of length  $g(s)$

8-neighbor grid

# A\* on Grid Graphs

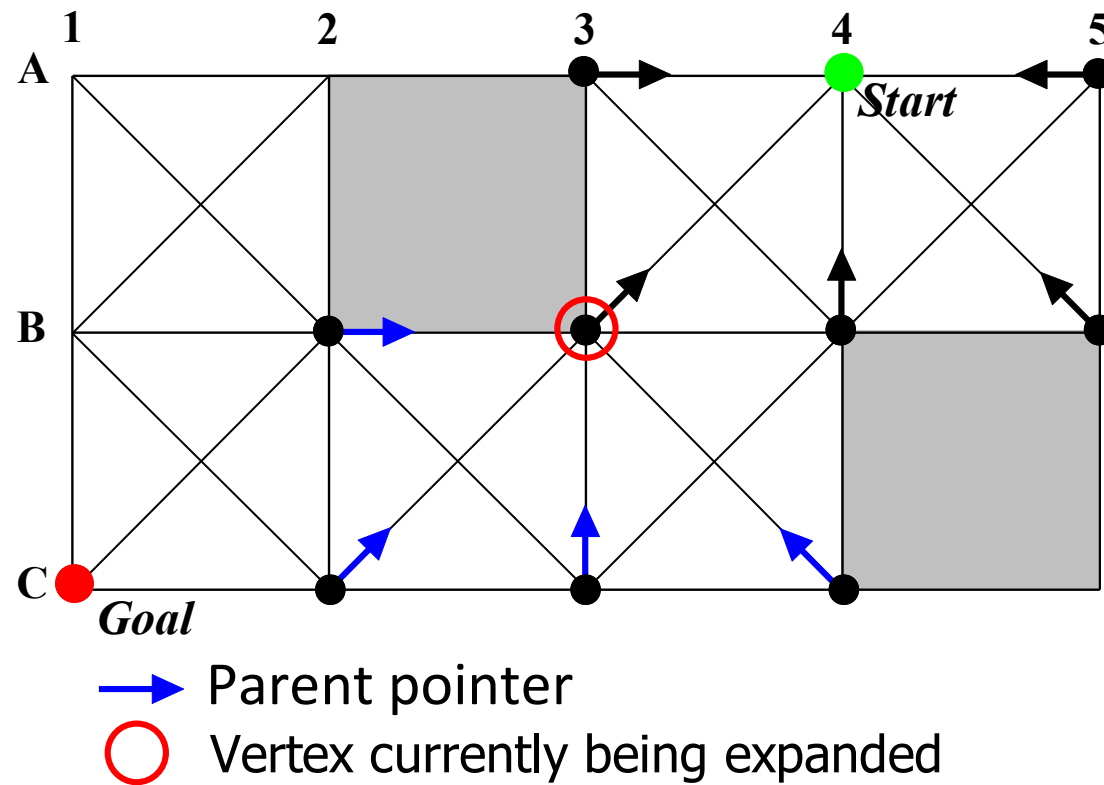
- A\* on grid graphs



8-neighbor grid

# A\* on Grid Graphs

- A\* on grid graphs

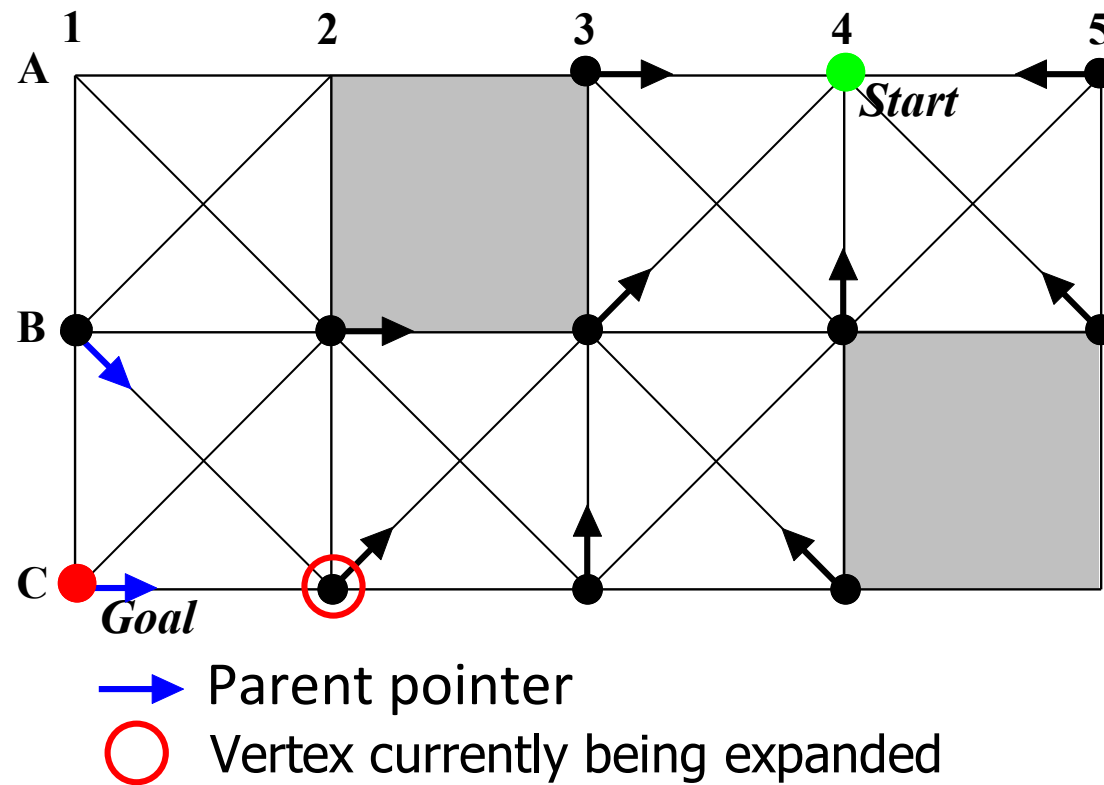


8-neighbor grid



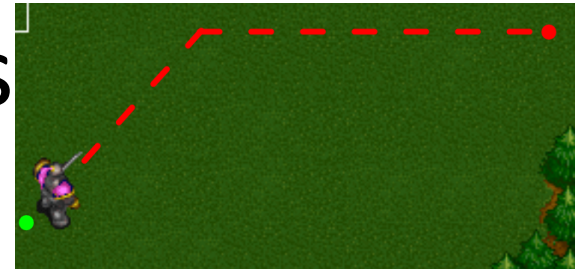
# A\* on Grid Graphs

- A\* on grid graphs

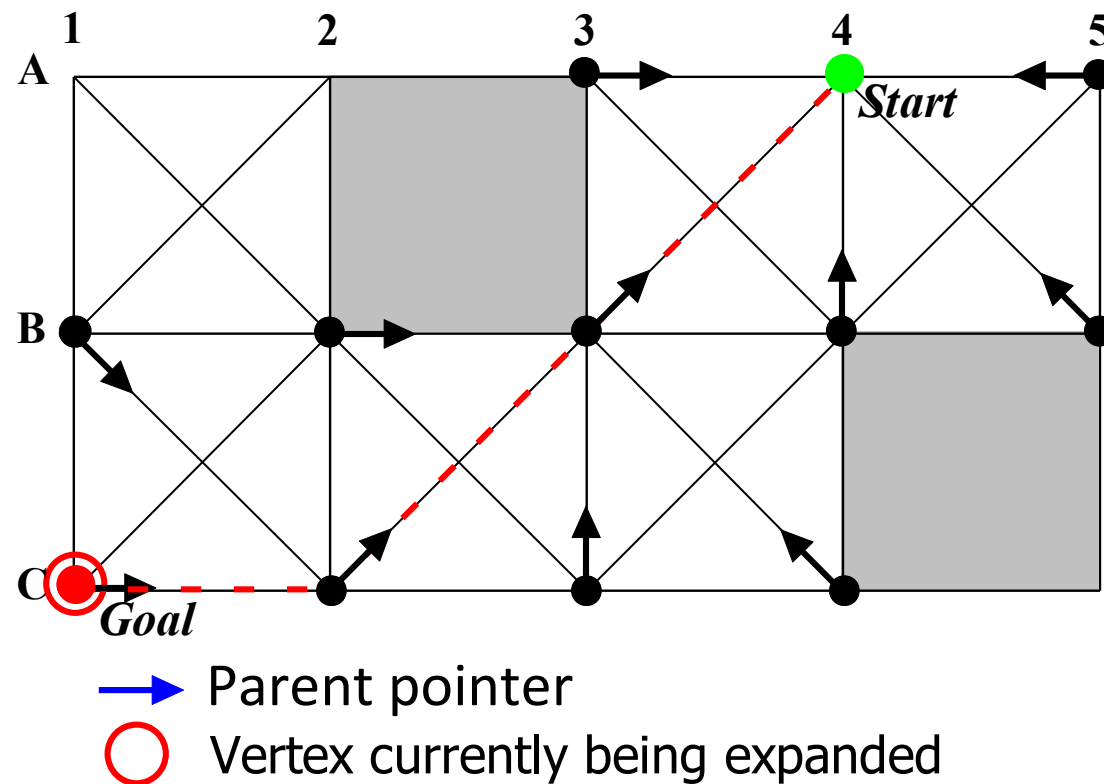


8-neighbor grid

# A\* on Grid Graphs

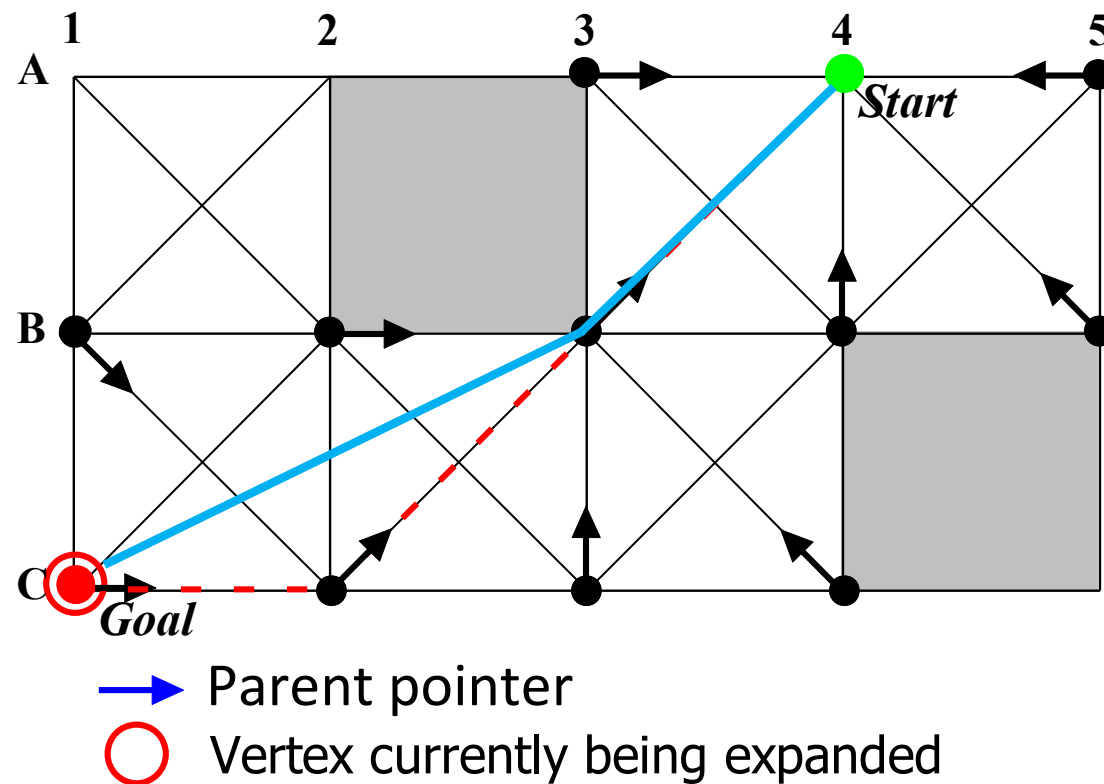


- A\* on grid graphs



# A\* on Grid Graphs

- A\* on grid graphs

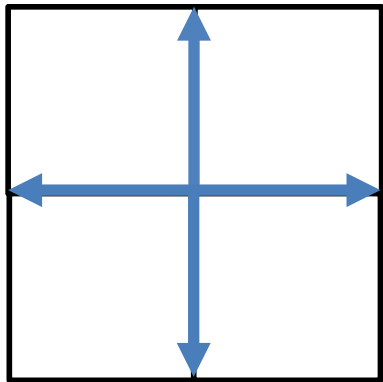


# A\* on Grid Graphs

Dimension	Regular Grid	Neighbors	% Longer Than Shortest Path
2D	triangular grid corners	3-neighbor	$\approx 100$
		6-neighbor	$\approx 15$
	square grid corners	4-neighbor	$\approx 41$
		8-neighbor	$\approx 8$
	hexagonal grid centers	6-neighbor	at least $\approx 15$
		12-neighbor	at least $\approx 4$
3D	cubic grid corners	6-neighbor	at least $\approx 73$
		26-neighbor	at least $\approx 13$

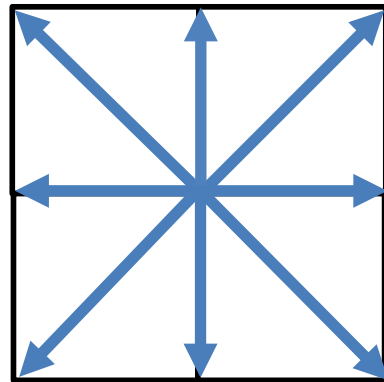
# Grids with Higher Degree Vertices

- Grid path finding on the  $2^k$  neighborhoods [Rivera et al.]



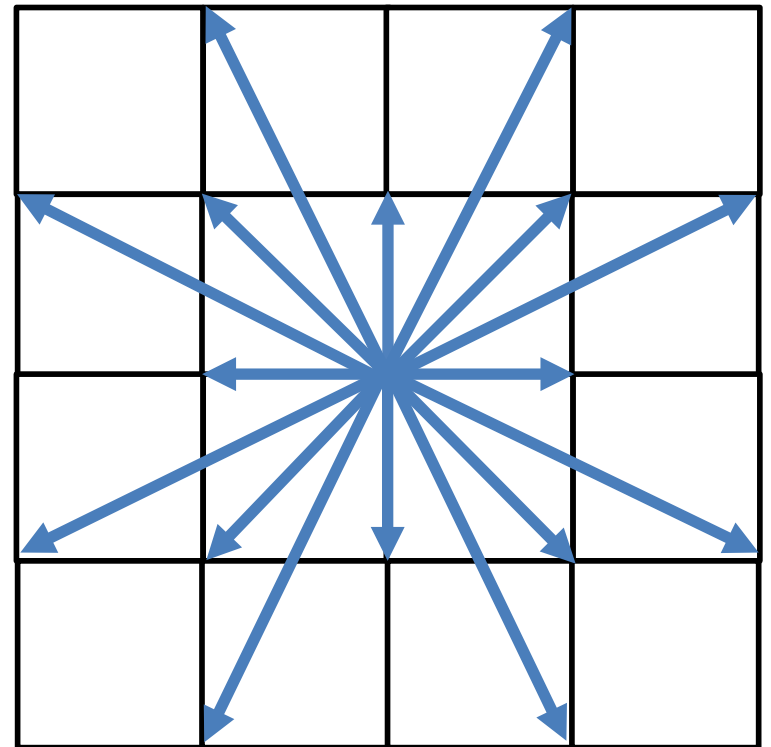
$$2^2=4$$

neighborhood



$$2^3=8$$

neighborhood

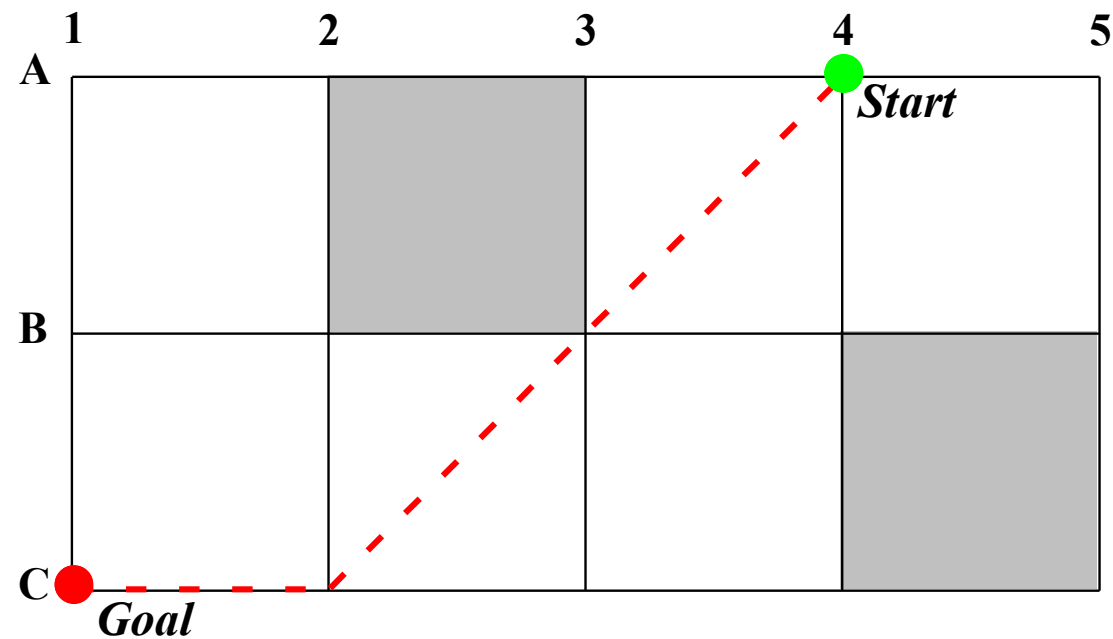


$$2^4=16$$

neighborhood

# A\* with Post Smoothing

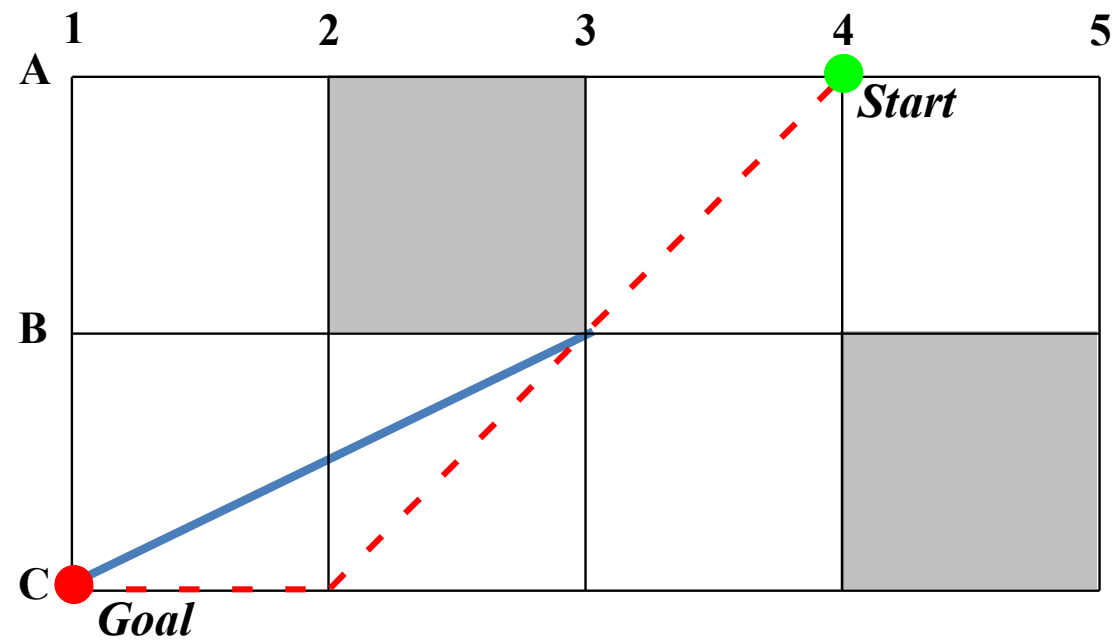
- A\* with Post Smoothing [Thorpe; Botea et al.; Millington]



8-neighbor grid

# A\* with Post Smoothing

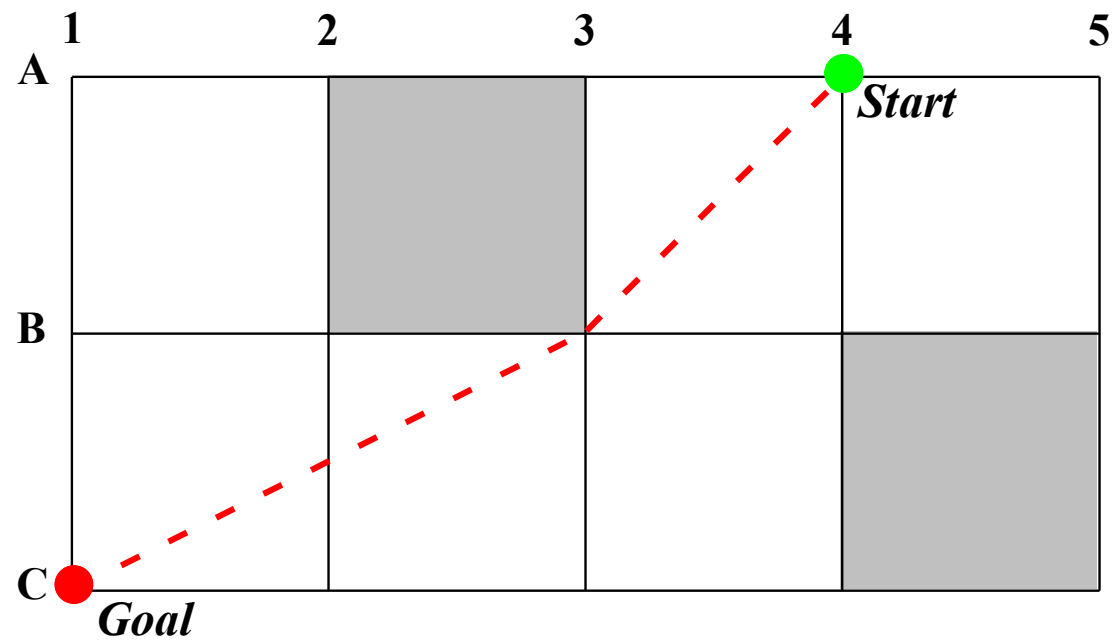
- A\* with Post Smoothing



8-neighbor grid

# A\* with Post Smoothing

- A\* with Post Smoothing

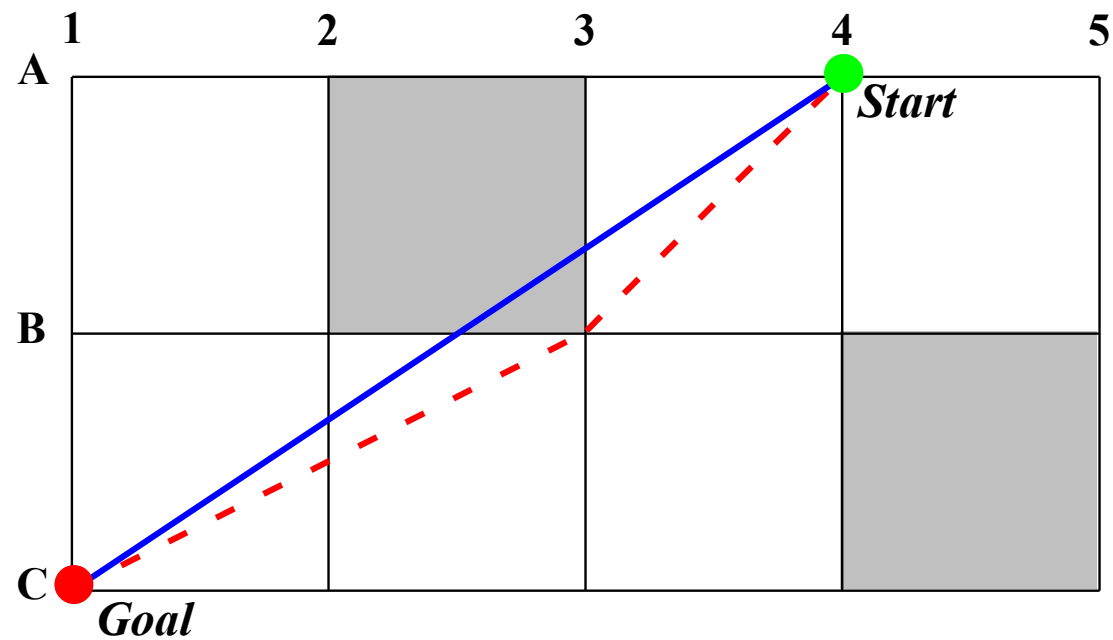


8-neighbor grid



# A\* with Post Smoothing

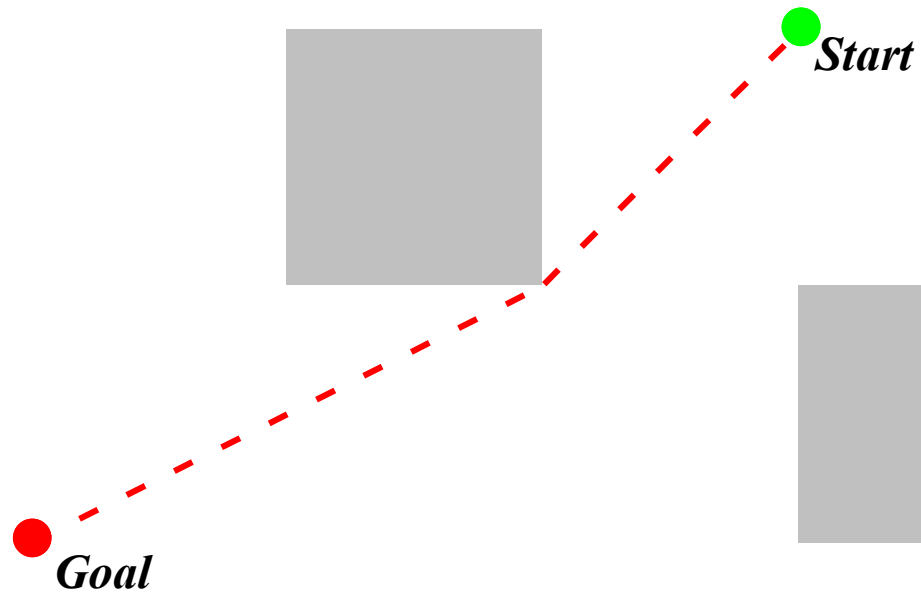
- A\* with Post Smoothing



8-neighbor grid

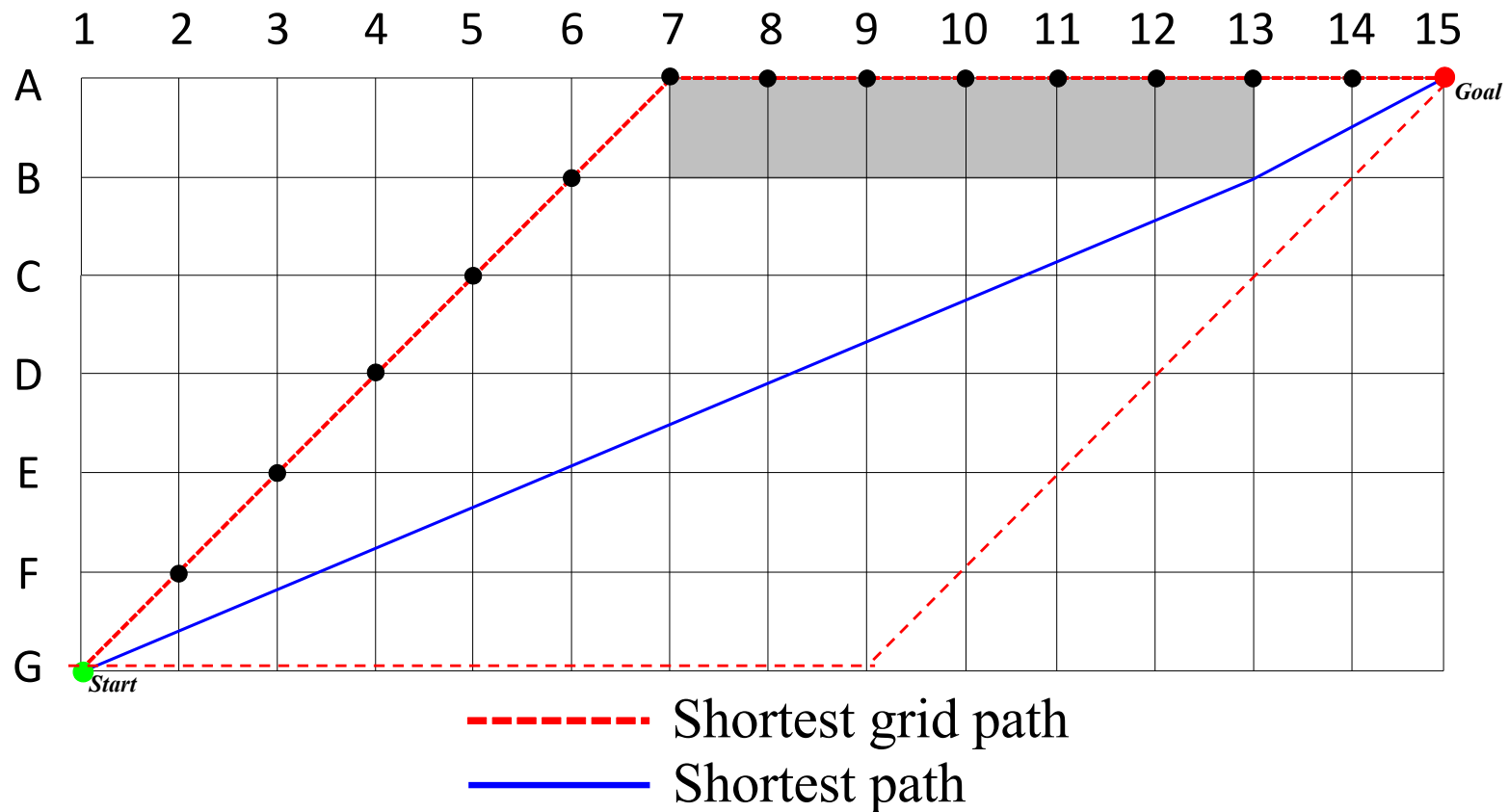
# A\* with Post Smoothing

- A\* with Post Smoothing



# A\* with Post Smoothing

- A\* with Post Smoothing



- Postprocessing often leaves path homotopy unchanged
- Better to interleave the search and the optimization

# Suboptimal Any-Angle Search

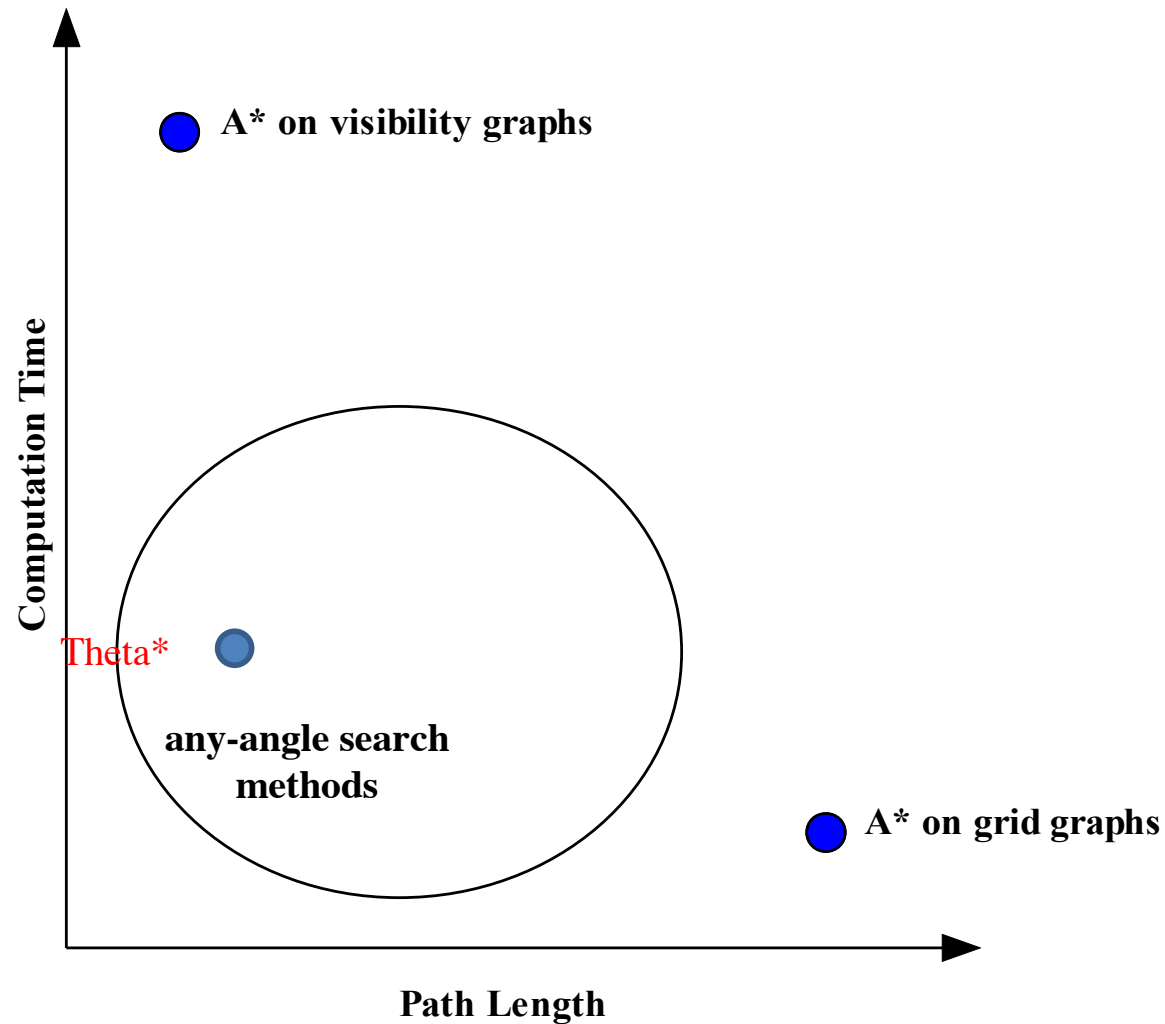
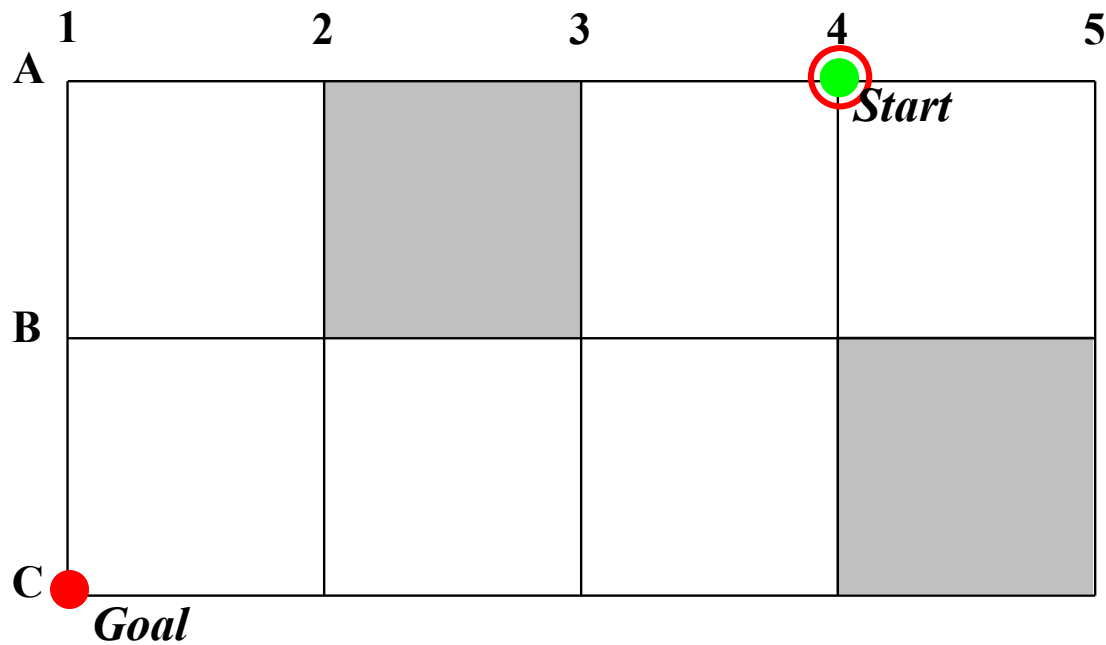


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# Suboptimal Theta\*

- Theta\*



Parent pointer

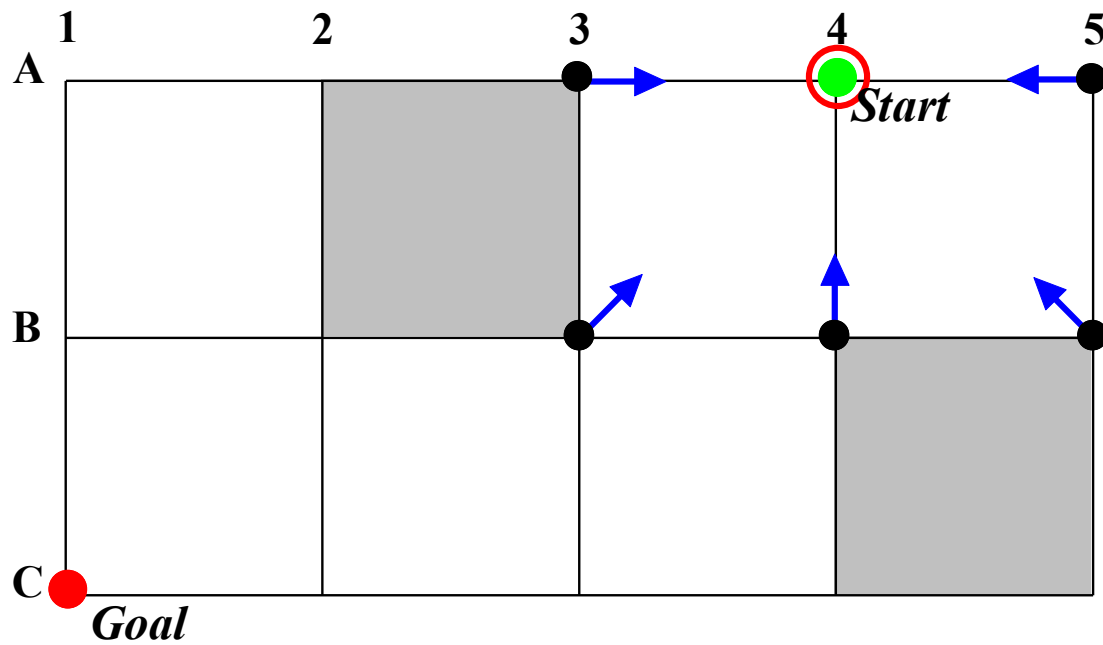


Vertex currently being expanded

8-neighbor grid

# Suboptimal Theta\*

- Theta\*



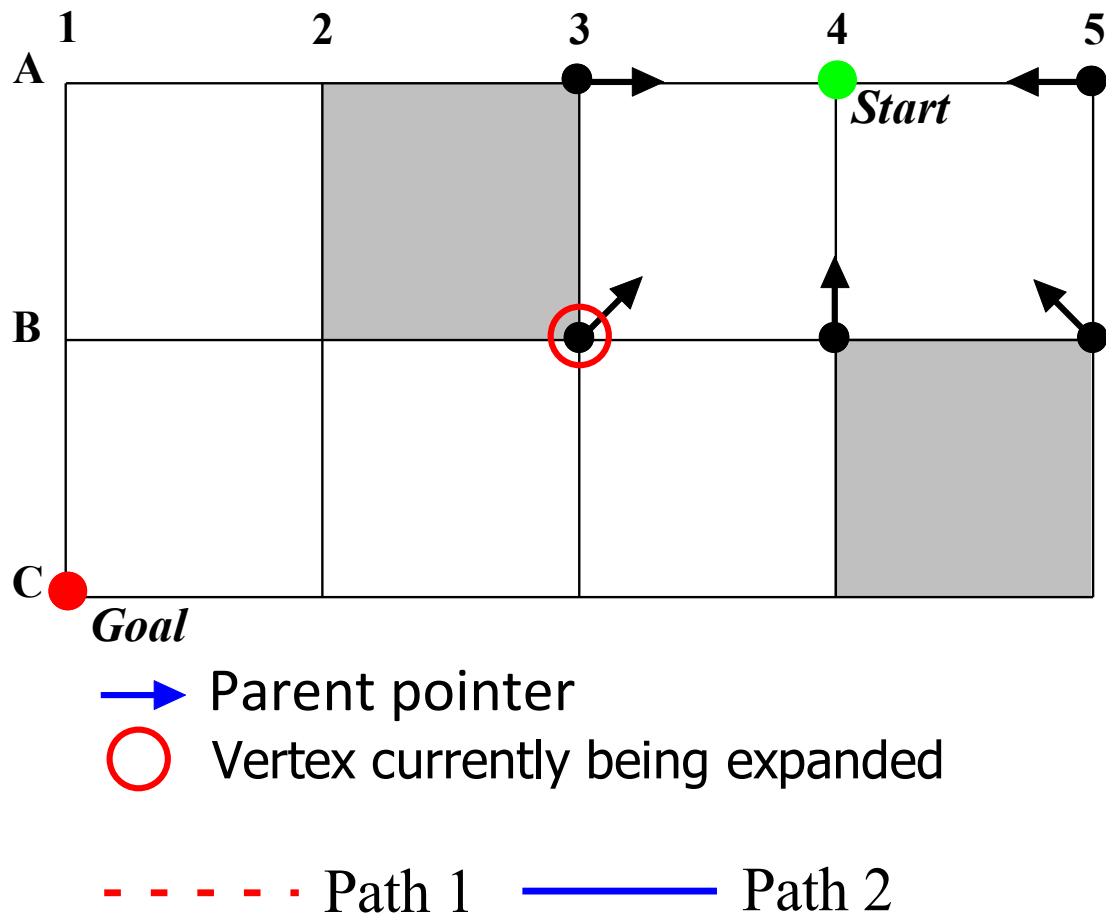
 Parent pointer  
 Vertex currently being expanded

- - - - Path 1      ——— Path 2

8-neighbor grid

# Suboptimal Theta\*

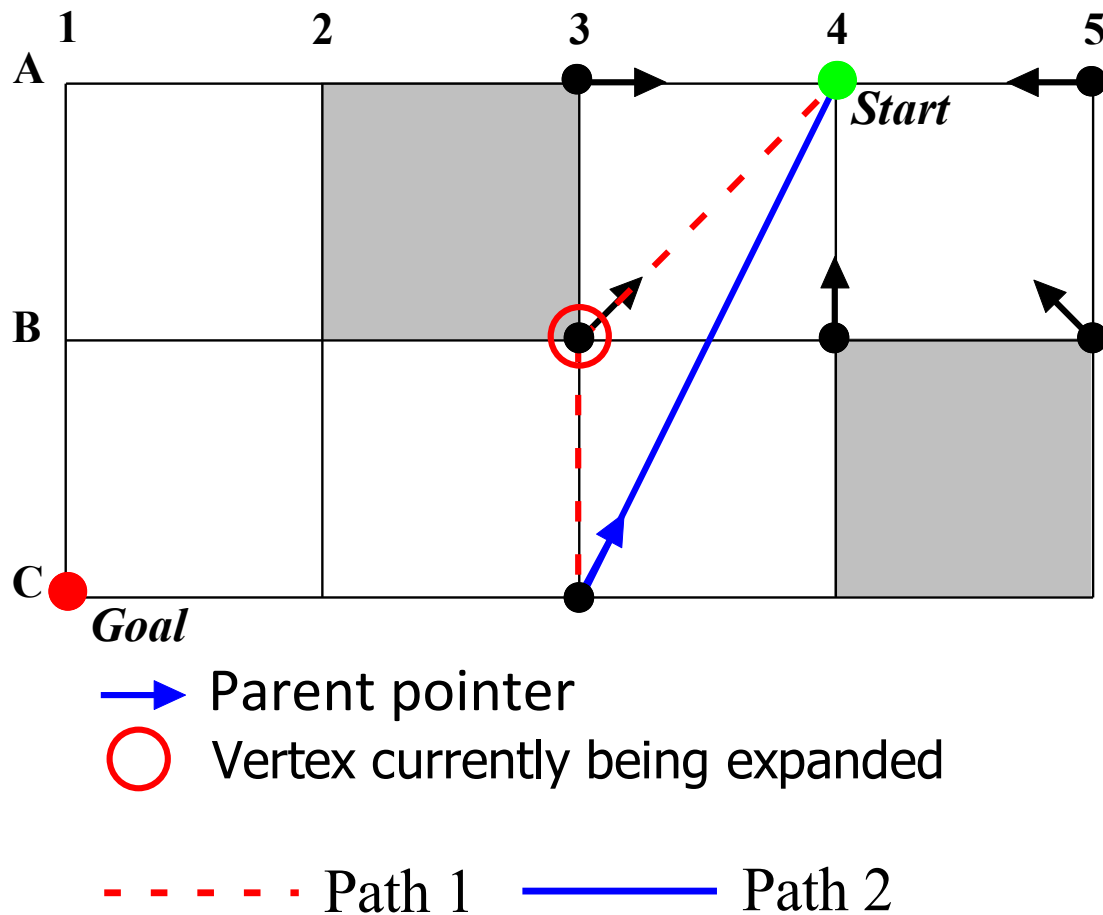
- Theta\*



8-neighbor grid

# Suboptimal Theta\*

- Theta\*

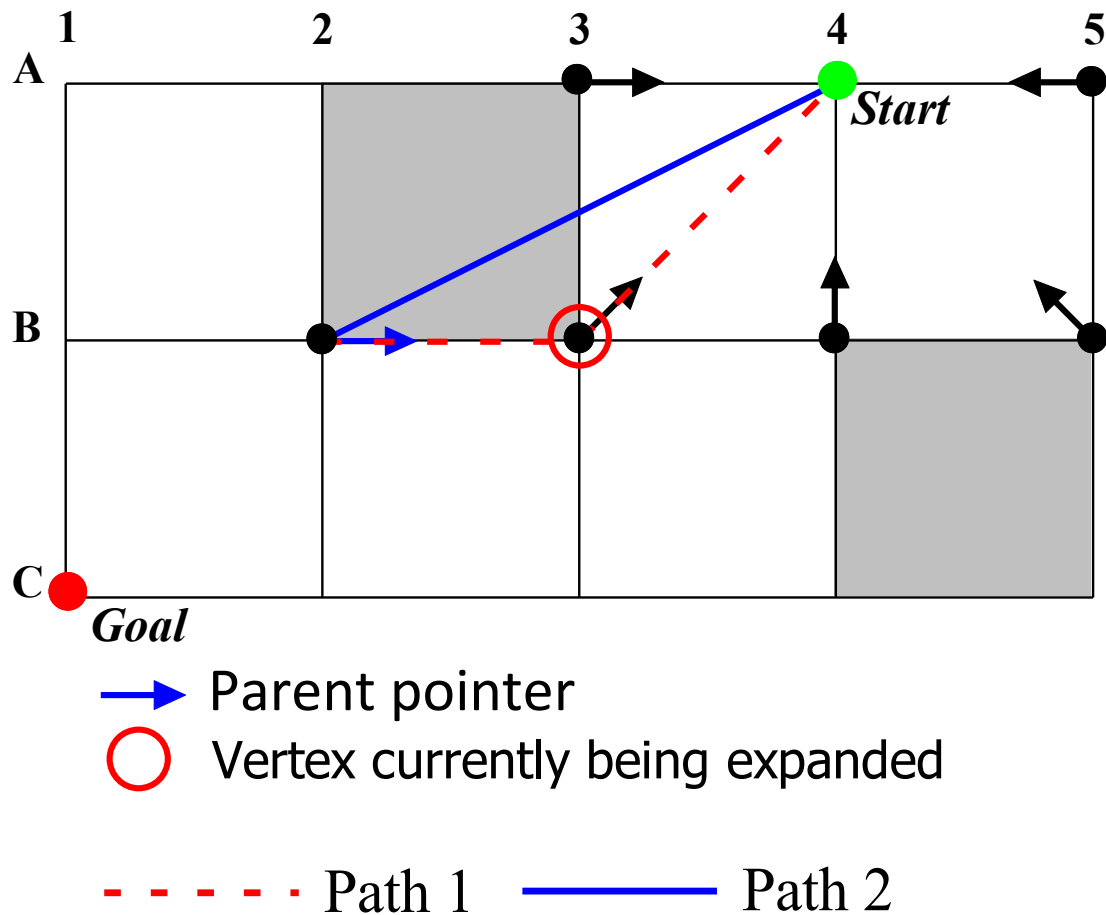


8-neighbor grid



# Suboptimal Theta\*

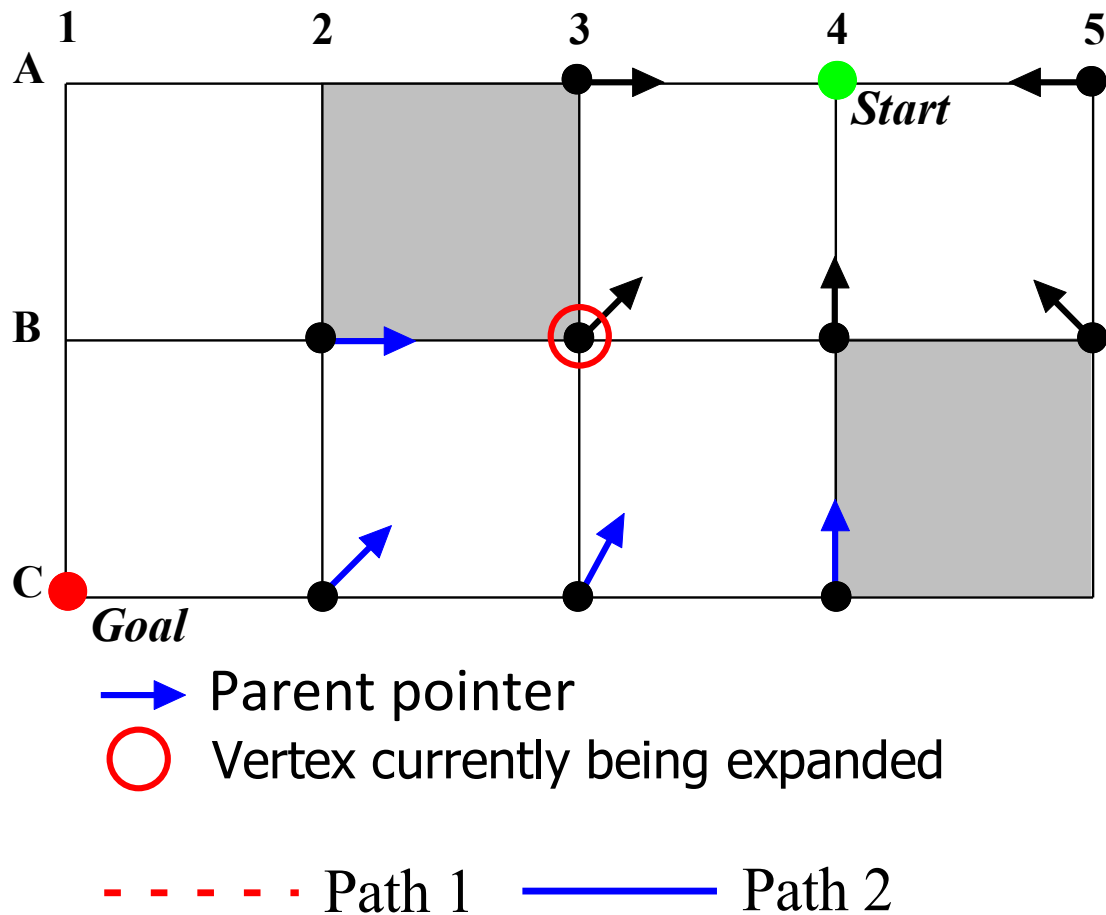
- Theta\*



8-neighbor grid

# Suboptimal Theta\*

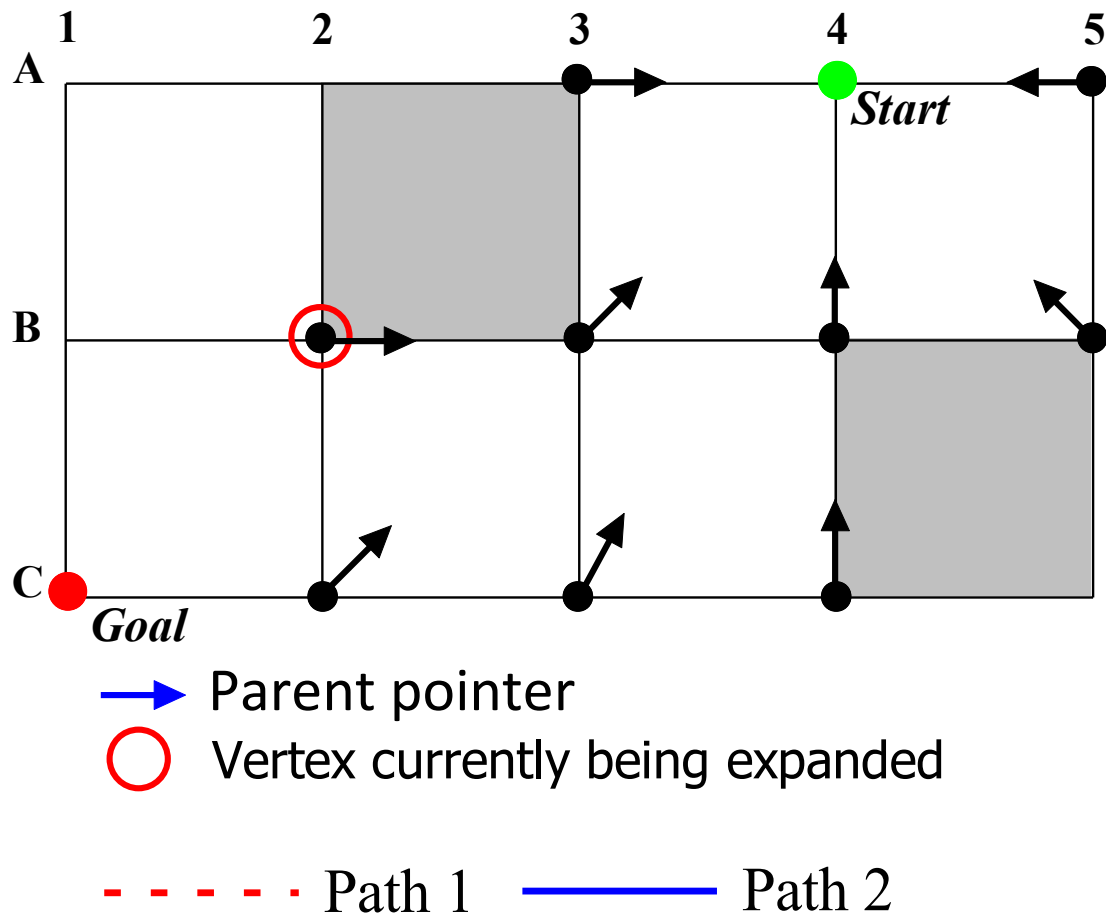
- Theta\*



8-neighbor grid

# Suboptimal Theta\*

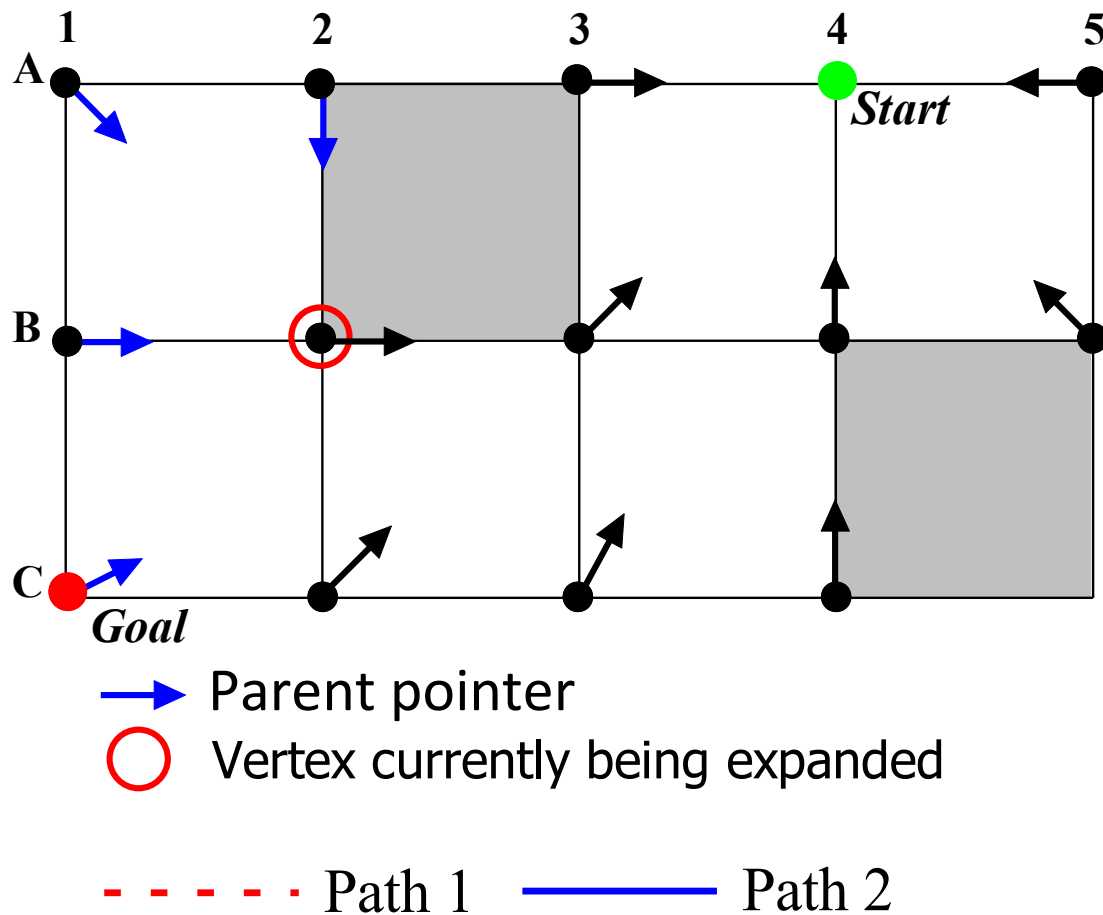
- Theta\*



8-neighbor grid

# Suboptimal Theta\*

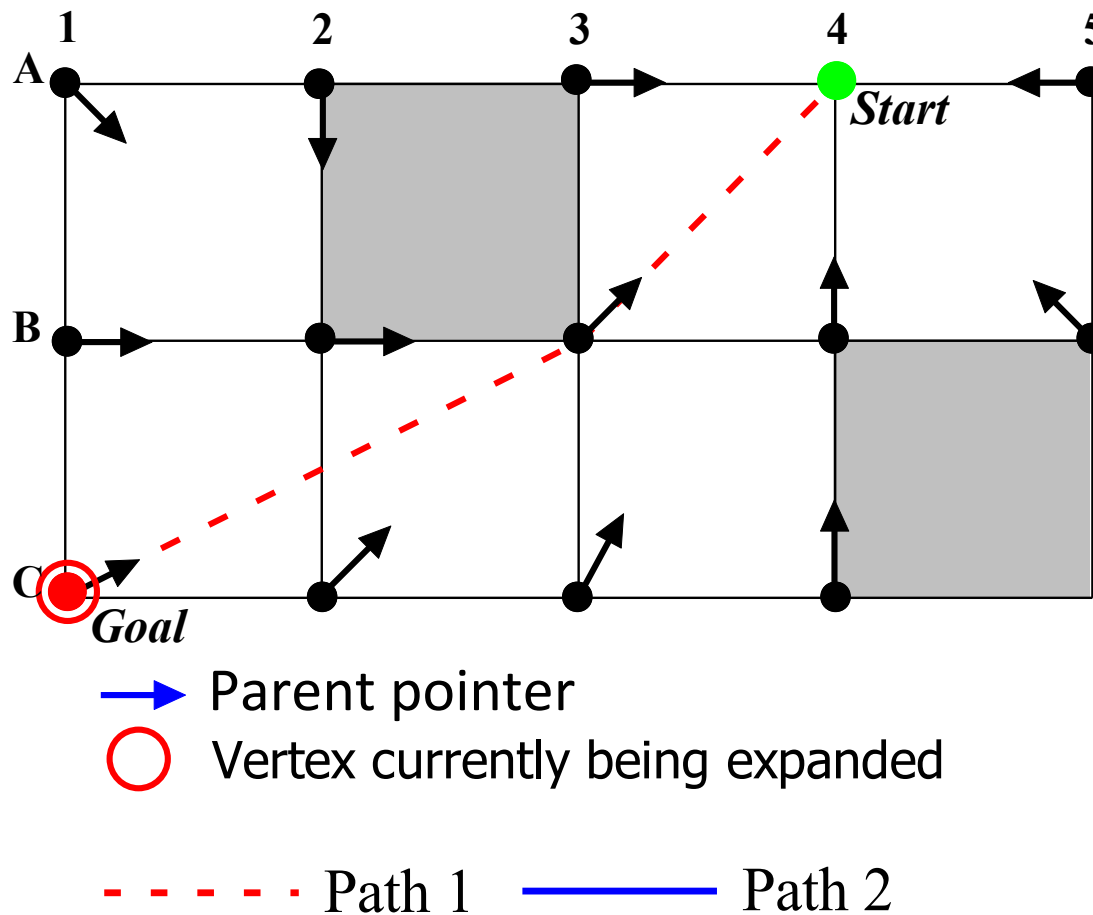
- Theta\*



8-neighbor grid

# Suboptimal Theta\*

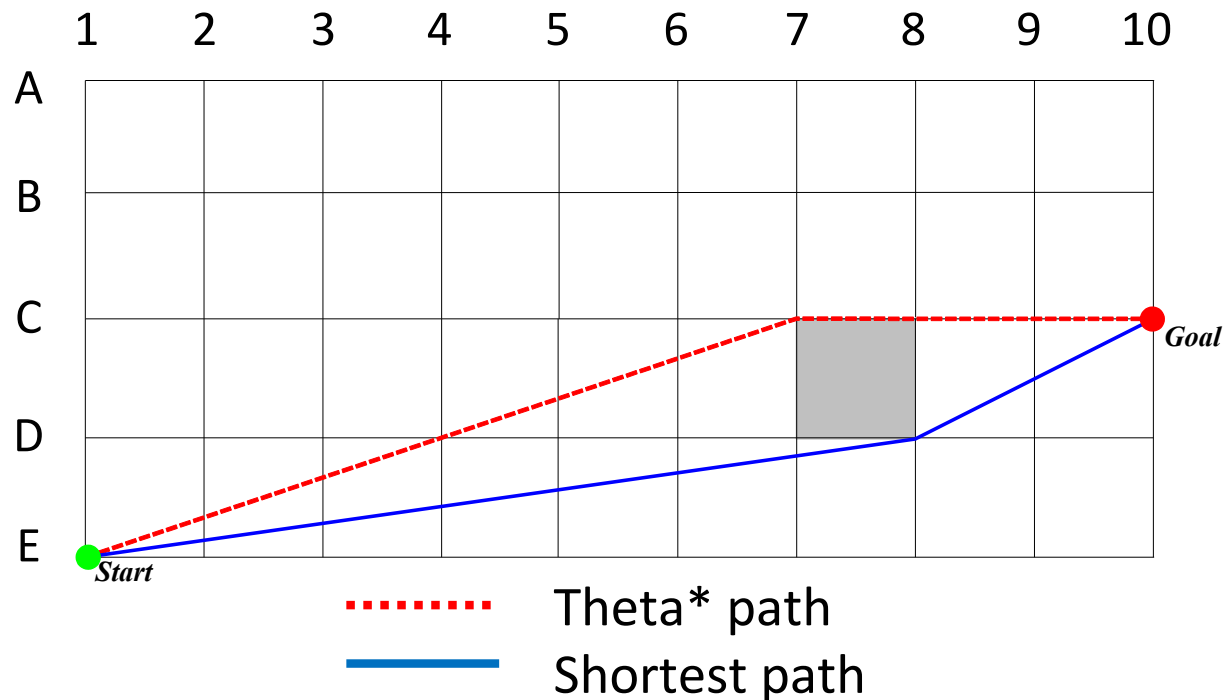
- Theta\*



8-neighbor grid

# Suboptimal Theta\*

- Theta\* is not guaranteed to find shortest paths since the parent of a vertex can only be a neighbor of the vertex or the parent of a neighbor



- The length of the path is still within 0.2% of optimal  
8-neighbor grid

# Suboptimal Lazy Theta\*

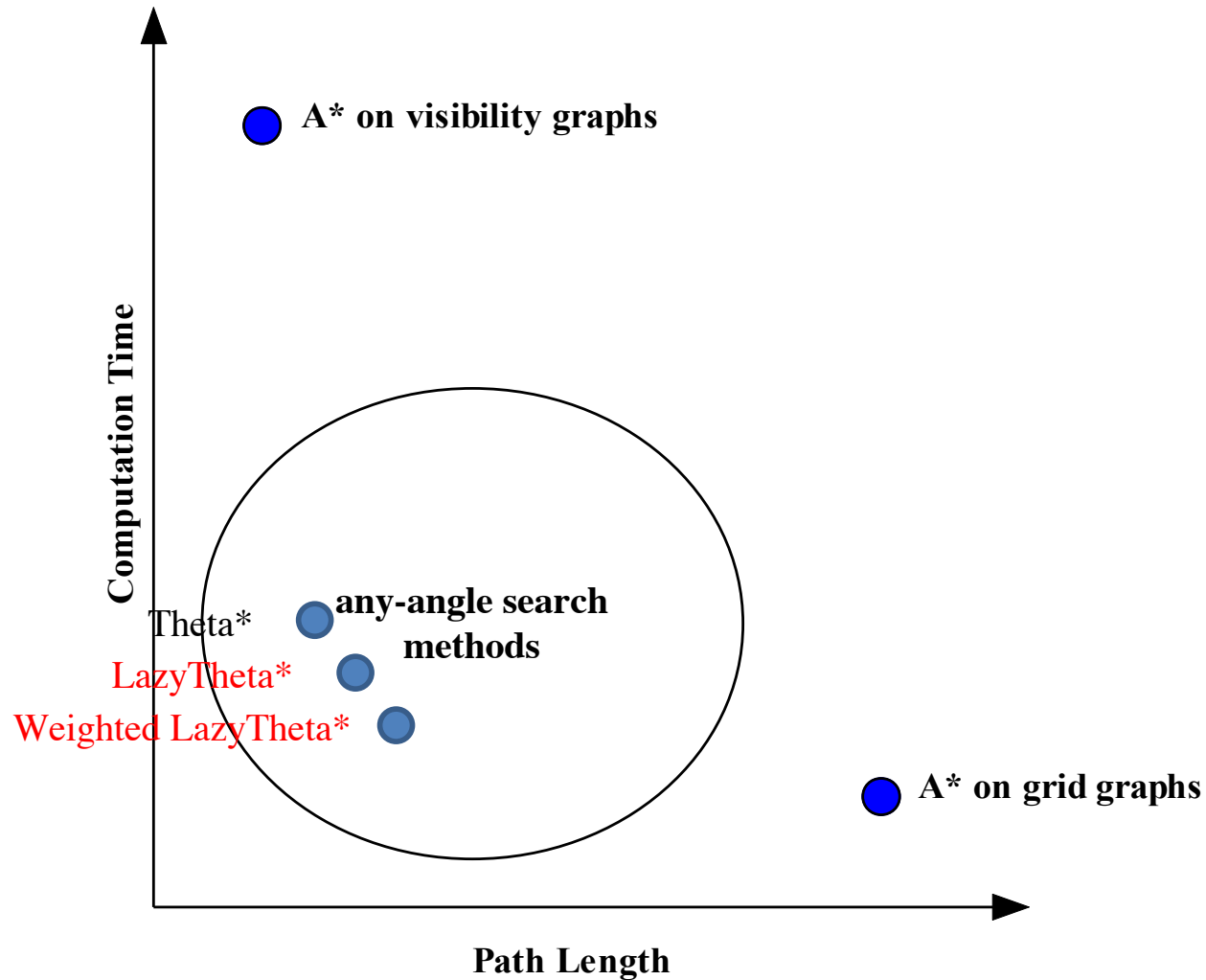
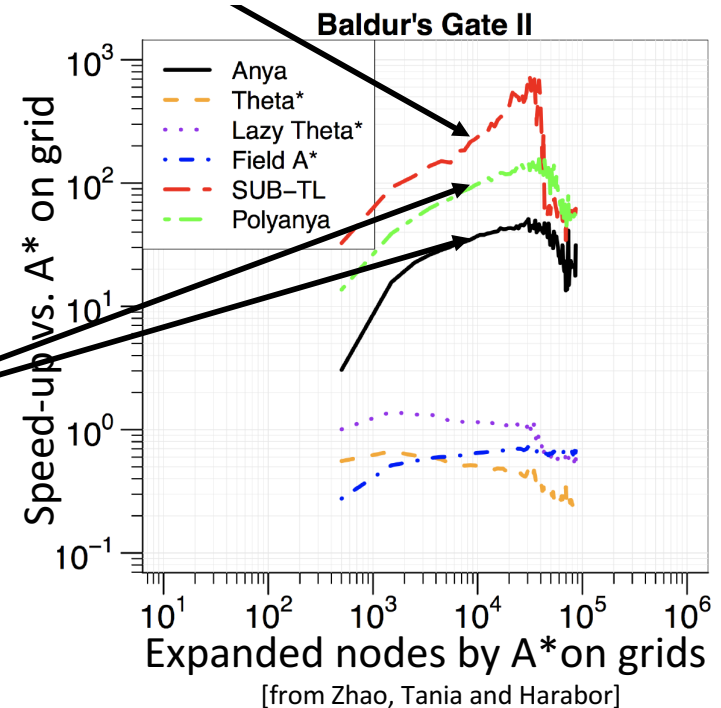
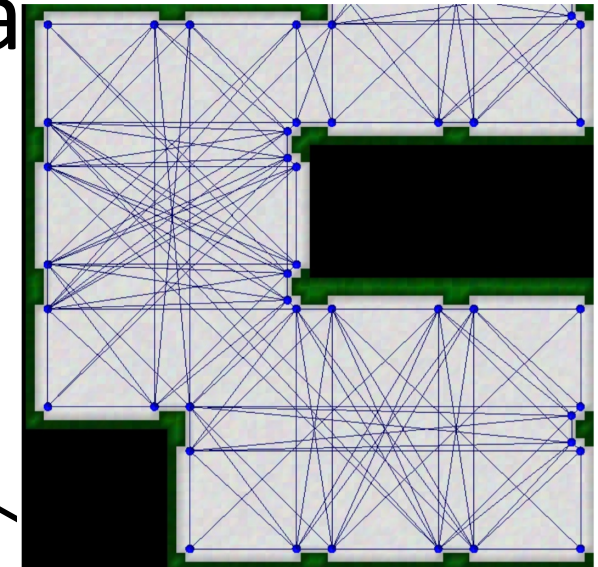


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# Alternatives to Theta\*

- Other any-angle search algorithms

- Several versions of Theta\*: Lazy Theta\*, Theta\* on Subgoal Graphs (SUB-TL)
- Accelerated A\* [Sislak et al.] a sophisticated version of Theta\*
- Field D\* [Ferguson and Stentz] an any-angle version of D\* (Lite) with interpolation
- Block A\* [Yap et al.] an any-angle version of A\* that operates on blocks of cells
- Anya and Polyanya [Harabor et al.] any-angle search methods for 2D that find shortest paths



[work done by different research groups]